Report problems for the first half of 'Systems Control Theory' 2020

Submit your report via Moodle system on ELMS (https://www.elms.hokudai.ac.jp/group/grouppage?idnumber=p20215501) by July 17, 2020. The report assignment consists of the following two questions.

Question 1: Consider a nonlinear system

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})u = \begin{pmatrix} 4x_1 + 3x_2 \\ -x_2(x_2^2+1) \end{pmatrix} + \begin{pmatrix} 1 - x_2^2 \\ (1+x_2^2)^2 \end{pmatrix} u,$$

where $\boldsymbol{x} = (x_1, x_2)^\top$ ($\in \mathbb{R}^2$) is a state vector and u ($\in \mathbb{R}$) is an input. We aim to perform the state-space exact linearization for the system. Answer the following questions.

(a) Show that $\zeta(\boldsymbol{x}) = x_1 - \dfrac{x_2}{x_2^2+1}$ is a solution of a partial differential equation $L_g\zeta = 0$.

(b) Obtain $(L_f\zeta)(\boldsymbol{x})$, $(L_gL_f\zeta)(\boldsymbol{x})$, and $(L_f^2\zeta)(\boldsymbol{x})$ in explicit forms.

(c) Show that $(L_gL_f\zeta)(\boldsymbol{x}) \neq 0$ for any $\boldsymbol{x}$.

(d) Obtain a feedback law $u = \alpha(\boldsymbol{x}, v)$ that exactly linearizes the system in the new state space $\boldsymbol{z} = \Phi(\boldsymbol{x}) = \begin{pmatrix} \zeta(x) \\ (L_f\zeta)(x) \end{pmatrix}$, where v is a new input variable.

Question 2: Consider a nonlinear system

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})u = \begin{pmatrix} x_1x_2 + \sin x_1 \\ -x_1^2 - 2x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u,$$
$$y = h(\boldsymbol{x}) = x_1 - x_2,$$

where $\boldsymbol{x} = (x_1, x_2)^\top$ ($\in \mathbb{R}^2$) is a state vector, u ($\in \mathbb{R}$) is an input, and y ($\in \mathbb{R}$) is an input. We will investigate a dissipativity of the system with a storage function $V(\boldsymbol{x}) = \dfrac{1}{2}(x_1^2 + x_2^2)$. Answer the following questions.

(a) Obtain $\dot{V}$ as a function of $\boldsymbol{x}$ and u.

(b) Show that $x_1 \sin x_1 \leq x_1^2$ for any x_1.

(c) Show that the system is OFP(-2) with the storage function $V(\boldsymbol{x})$.

(d) Show that the system can be globally asymptotically stabilized by a feedback $u = -ky$, where k is a constant greater than 2.