

Detailed information in numerical examples

First, Boolean functions of a CS-PBN to be considered in numerical examples are given as follows.

$$f^{(1)} = \begin{cases} f_1^{(1)} = x_1(k) \wedge \neg x_6(k) \vee u_3(k), \\ c_1^{(1)} = 0.8, \\ f_2^{(1)} = x_2(k) \wedge x_4(k) \wedge \neg x_8(k), \\ c_2^{(1)} = 0.2, \end{cases}$$

$$f^{(2)} = \begin{cases} f_1^{(2)} = \neg x_4(k) \wedge u_1(k) \vee u_3(k), \\ c_1^{(2)} = 0.7, \\ f_2^{(2)} = \neg x_2(k) \wedge x_3(k) \vee u_3(k), \\ c_2^{(2)} = 0.3, \end{cases}$$

$$f^{(3)} = \begin{cases} f_1^{(3)} = x_5(k) \wedge u_1(k) \vee \neg x_{10}(k) \wedge x_{12}(k) \\ \wedge u_3(k), c_1^{(3)} = 0.6, \\ f_2^{(3)} = x_1(k) \vee \neg x_2(k) \wedge x_3(k) \wedge x_4(k), \\ c_2^{(3)} = 0.4, \end{cases}$$

$$f^{(4)} = \begin{cases} f_1^{(4)} = x_2(k) \wedge x_5(k) \wedge \neg u_1(k) \\ \vee \neg x_{14}(k), c_1^{(4)} = 0.9, \\ f_2^{(4)} = \neg x_1(k) \wedge x_2(k) \wedge u_1(k) \vee x_{14}(k), \\ c_2^{(4)} = 0.1, \end{cases}$$

$$f^{(5)} = \begin{cases} f_1^{(5)} = \neg u_1(k) \wedge x_6(k) \wedge x_7(k) \vee x_{12}(k) \\ \wedge x_{14}(k), c_1^{(5)} = 0.7, \\ f_2^{(5)} = \neg u_2(k) \wedge x_{13}(k) \wedge x_{14}(k) \\ \wedge x_{15}(k), c_2^{(5)} = 0.3, \end{cases}$$

$$f^{(6)} = \begin{cases} f_1^{(6)} = x_1(k) \wedge x_6(k) \wedge x_{10}(k) \vee \neg x_{15}(k), \\ c_1^{(6)} = 0.5, \\ f_2^{(6)} = \neg x_2(k) \vee x_5(k) \wedge u_1(k) \wedge \neg u_3(k), \\ c_2^{(6)} = 0.5, \end{cases}$$

$$f^{(7)} = \begin{cases} f_1^{(7)} = x_6(k) \wedge x_7(k) \wedge x_8(k) \vee u_2(k) \\ \wedge \neg u_3(k), c_1^{(7)} = 0.8, \\ f_2^{(7)} = u_1(k) \wedge x_{13}(k), c_2^{(7)} = 0.2, \end{cases}$$

$$f^{(8)} = \begin{cases} f_1^{(8)} = x_5(k) \wedge \neg u_1(k) \vee x_{10}(k) \wedge u_2(k) \\ \wedge x_{13}(k), c_1^{(8)} = 0.7, \\ f_2^{(8)} = x_5(k) \wedge x_{13}(k), c_2^{(8)} = 0.3, \end{cases}$$

$$f^{(9)} = \begin{cases} f_1^{(9)} = x_3(k) \wedge u_1(k) \vee \neg x_8(k) \wedge x_{11}(k), \\ c_1^{(9)} = 0.8, \\ f_2^{(9)} = \neg x_6(k), c_2^{(9)} = 0.2, \end{cases}$$

$$f^{(10)} = \begin{cases} f_1^{(10)} = x_6(k), c_1^{(10)} = 0.6, \\ f_2^{(10)} = x_2(k) \wedge u_2(k) \wedge \neg x_{12}(k) \wedge u_3(k), \\ c_2^{(10)} = 0.4, \end{cases}$$

$$f^{(11)} = \begin{cases} f_1^{(11)} = x_6(k) \wedge x_{10}(k) \vee \neg u_2(k) \wedge u_3(k), \\ c_1^{(11)} = 0.6, \\ f_2^{(11)} = \neg x_5(k) \wedge u_1(k) \vee \neg x_{15}(k) \\ \wedge u_3(k), c_2^{(11)} = 0.4, \end{cases}$$

$$f^{(12)} = \begin{cases} f_1^{(12)} = x_{12}(k) \wedge \neg x_{15}(k), c_1^{(12)} = 0.9, \\ f_2^{(12)} = x_7(k) \wedge x_3(k), c_2^{(12)} = 0.1, \end{cases}$$

$$f^{(13)} = \begin{cases} f_1^{(13)} = \neg u_3(k), c_1^{(13)} = 0.7, \\ f_2^{(13)} = x_7(k) \wedge u_2(k) \wedge u_3(k), \\ c_2^{(13)} = 0.3, \end{cases}$$

$$f^{(14)} = \begin{cases} f_1^{(14)} = \neg x_{14}(k) \wedge u_3(k), c_1^{(14)} = 0.8, \\ f_2^{(14)} = x_{12}(k) \vee x_{14}(k) \wedge u_3(k), \\ c_2^{(14)} = 0.2, \end{cases}$$

$$f^{(15)} = \begin{cases} f_1^{(15)} = x_{14}(k) \wedge x_{15}(k), c_1^{(15)} = 0.5, \\ f_2^{(15)} = x_8(k), c_2^{(15)} = 0.5. \end{cases}$$

Next, $q(k)$ in a CS-PBN is given as

$$q(0) = 0.8, \quad q(1) = 0.7, \quad q(2) = 0.6, \quad q(3) = 0.8, \\ q(4) = 0.9, \quad q(5) = 0.6, \quad q(6) = 0.5, \quad q(7) = 0.7, \\ q(8) = 0.6, \quad q(9) = 0.9.$$

Finally, the weighting vectors in Problem A and Problem B are given as

$$Q = [1 \ 1 \ \dots \ 1], \\ R = [1 \ 0 \ 1], \\ Q_f = [10 \ 10 \ \dots \ 10].$$

The initial state is given as $x_0 = [1 \ 1 \ \dots \ 1]^T$.